

**CS 03**

**Investigating the Effectiveness of  
Different Control Algorithms on the  
Stability of Quadcopters**

**Gao Wen Zhen, James 19S7J  
Roy Chenyu Luo 19S7J**

**Mr Samuel Tan**

## 1. Background and Purpose of Research

Unmanned Aerial Vehicles (UAVs) are defined as aircraft without any onboard presence of pilots (Gene et al., 1997). Recently, UAVs of the quadcopter variety has risen in popularity due to its versatility and affordability. It has been adapted for many different uses, including both military and civilian usages, such as air quality monitoring, area mapping, and surveillance. Fixed-wing UAV designs are comparatively more efficient, but they need a sizable air strip for landing and takeoff, which is a problem in land-scarce Singapore (Thamm et al., 2015). Quadcopters, in comparison, are able to achieve vertical takeoff and landing (VTOL), which is ideal for this land-scarce situation.

For the quadcopter's various applications, a stable and responsive control system is a prerequisite. Unlike fixed-wing aircraft, quadcopters operate by two pairs of identical propellers; two clockwise and two counterclockwise. By changing the speed of the motors, a desirable total thrust is generated from the propellers, allowing it to orientate itself in 3-dimensional space. A multirotor platform is aerodynamically unstable due to the lack of inherent lift generating surfaces, thus is impossible to fly in a fully open-loop system (Stafford, 2014). Therefore, quadcopters need to have a regular estimate of its orientation in 3-dimensional space, in order to make adjustments through its actuators. This is done by using an Inertial Measurement Unit (IMU) that measures various data that can be utilized to estimate the platform's orientation.

Aircraft convention defines that the orientation of a body can be represented in terms of roll, pitch, and yaw angles (Figure 1). These angles are relative to the horizontal plane which is perpendicular to the direction of gravity at that particular location. Roll is defined as the angle between the body's lateral axis and the horizontal plane. Pitch is defined as the angle between the body's longitudinal axis and the horizontal plane. Yaw is the rotation made around an axis parallel to the local gravitational vector. These Euler angles are the necessary inputs used in most calculations in the control system, which returns modified output values to the actuators to ensure stable flight.

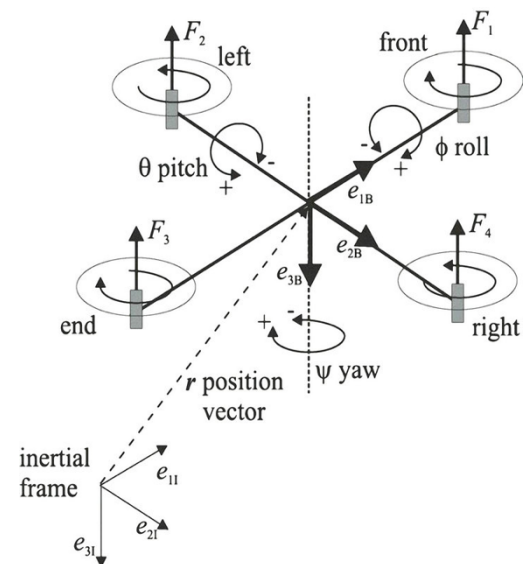


Figure 1: Aircraft conventions

Previously, research has identified many possible control systems such as a Linear Quadratic Gaussian (LQG) controller design (Fessi & Bouallegue, 2016) and a fuzzy logic controller (Doitsidis et al., 1997). The most popular, the Proportional-Integral-Derivative (PID) control method, is preferred by many for its simplicity and intuitiveness. (Kada & Ghazzawi, 2011)

The PID controller is a closed feedback mechanism that alters the output dynamically according to the previous state of the system.

In every loop, the flight controller will compute the error term  $e(t)$  which is the difference between the setpoint  $r(t)$  and the process variable  $y(t)$ . As seen in Figure 2,  $e(t)$  will be passed through the PID controller before producing a control variable  $u(t)$  that will be passed to the plant process, which in our case is to change the speed of each motor. After this, a new  $y(t)$  value is measured by the IMU after the motors have changed the orientation of the multirotor. This process repeats until the goal of reducing the error to zero is achieved and the desired orientation is obtained. It uses the three control terms of proportional, integral and derivative to apply accurate and optimal control.

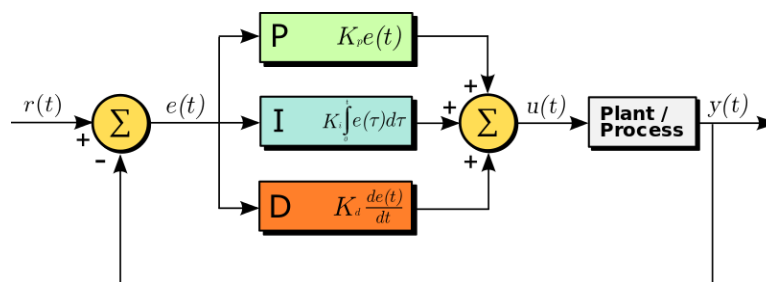


Figure 2: A block diagram of a PID controller in a closed feedback loop

The proportional term produces an output that is proportional to the current error. The integral term takes the sum of all the previous error terms, allowing any residual error to be accounted for, eliminating any steady-state error that may be present. The derivative term produces an output based on its current rate of change in error to ensure that the system reaches the setpoint smoothly.

$$u(t) = K_p \cdot e(t) + K_i \int_0^t e(t) dt + K_d \frac{de(t)}{dt}$$

Above is the mathematical form of the PID controller which can be simply implemented in code (Arduino) with a few lines.  $K_p$ ,  $K_i$  and  $K_d$  are constants or gains that have to be tuned carefully to ensure the stability of the quadcopter. Each quadcopter will have different gains due to different physical layouts and aerodynamic responsiveness.

The inputs of the system can be varied by taking the  $e(t)$  term as the angle the quadcopter is at (angles controller), or the angular velocity of the drone (rates controller). The output of the system are variable time pulses sent to the electronic speed controllers (ESCs) which change the speed of the 4 motors of which will orient the quadcopter.

## 2. Objectives

This project aims to investigate the effect of changing the gains of the PID terms on the accuracy and latency of the angles controller, as well as comparing the accuracy and latency between the rates controller and the angles controller.

Accuracy can be defined, in our case, to be the degree to which the result of the angle the drone makes to the horizontal conforms to the setpoint, which is  $0^\circ$ .

Latency can be defined as the delay before reaching the set-point, following a deviation. The lower the latency, the faster the correction is able to be made.

### 3. Materials and Methods

#### 3.1 Microcontroller and IMU

The microcontroller used was the Teensy 3.5. The orientation estimation of the flight controller was computed by an IMU. The breakout board, GY-82, consisted of a 3-axis gyroscope and a 3-axis accelerometer.

##### 3.2.1 Gyroscope

A gyroscope measures a body's angular velocity around its 3 axes of rotation. With that, the roll ( $\theta$ ), pitch ( $\phi$ ) and yaw ( $\psi$ ) angles are simply the angular displacement in each axis which can be determined by integrating the angular velocity ( $\omega$ ) of their respective axes over time. In other words, the summation of the products of the angular velocity and the predefined time step ( $\Delta t$ ) of  $4000\mu s$ . The subscript  $k$  signifies the previous reading.

$$\theta = \theta_k + \omega_x \Delta t$$

$$\phi = \phi_k + \omega_y \Delta t$$

$$\psi = \psi_k + \omega_z \Delta t$$

However, inherent imperfections and noise within the gyroscope will cause the integrated value of the angular velocity to drift over time, not returning to zero after the body is in its original position. This results in an unreliable orientation estimation which can cause the multirotor to unnecessarily correct its orientation. Therefore, sensor fusion with an accelerometer is used.

##### 3.2.2 Accelerometer

An accelerometer measures all forces acting on the sensor. Thus, the accelerometer also measures gravitational acceleration together with linear acceleration. For orientation estimation, it is assumed that there is negligible linear acceleration. With that, the accelerometer is essentially a gravity sensor that is capable of detecting the direction of the local gravitational vector. A 3-axis accelerometer was used, which measures the local

gravitational vector in 3 different axes ( $a_x, a_y, a_z$ ) and the sum of the squares of each accelerometer reading will be equivalent to 1g.

$$\sqrt{a_x^2 + a_y^2 + a_z^2} = 1g$$

According to Freescale Semiconductor (Pedley, 2013) , the pitch and roll angles can be computed with the following equations after applying rotation matrices to the gravitational vector:

$$\tan(\theta) = \frac{a_y}{a_z}$$

$$\tan(\phi) = -\frac{a_x}{\sqrt{a_y^2 + a_z^2}}$$

### 3.3 Experimental Setup



Figure 3 shows the experimental setup where the quadcopter was secured to a well-lubricated rod, allowing freedom in the roll axis. This allowed us to isolate and compare the effect of the PID values on a single axis.

Figure 3: Quadcopter secured to a rod allowing for rotation about a single axis

## 4. Results and Discussion

### 4.1 Comparison of P-gain on the stability of quadcopter

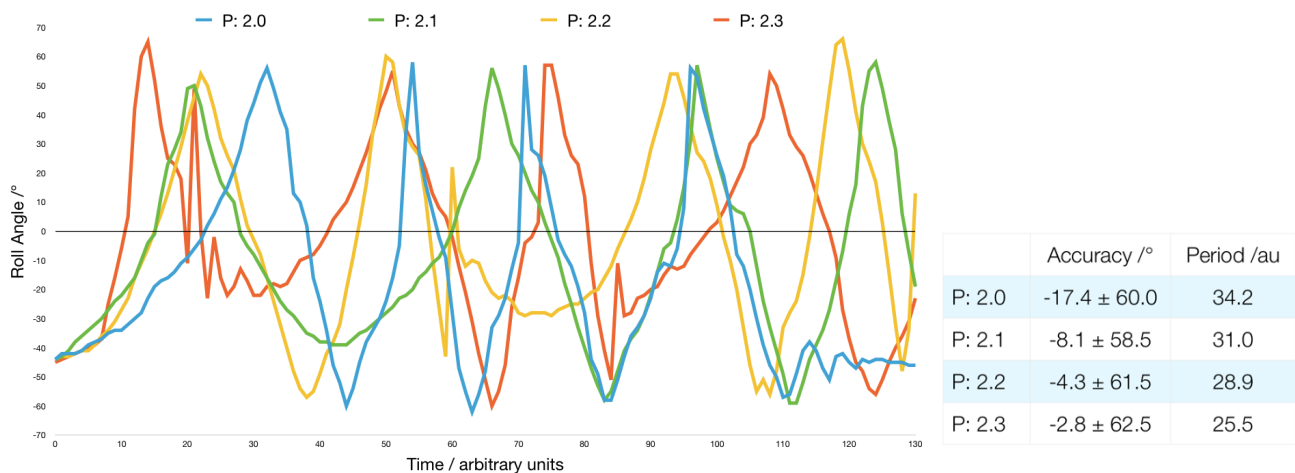


Figure 4: Comparison of different P-gain values on the stability of the quadcopter

As observed by the data recorded in the figure above, the quadcopter oscillated about the setpoint of 0°. As the P gain increased from 2.0 to 2.3, the period of the oscillation increases. As the P gain increased, the accuracy did too.

As the proportional term produces an output proportional to the current error, when the P-gain increases, the quadcopter reacts more aggressively to errors due to larger motor correction output values. Thus, the quadcopter reacts faster to changes and becomes faster.

However, the accuracy did not increase as the quadcopter continued to overshoot and hit  $\pm 60^\circ$ . As the larger motor correction values created a correcting force, which did not decrease until after the quadcopter has reached the setpoint, thus the inertia of this correcting force caused the quadcopter to overshoot and overcorrect. This is expected, and can be resolved by an appropriate D-gain, which is able to act as “resistance” to the motion, as explained below.

#### 4.2 Comparison of I-gain on the stability of quadcopter

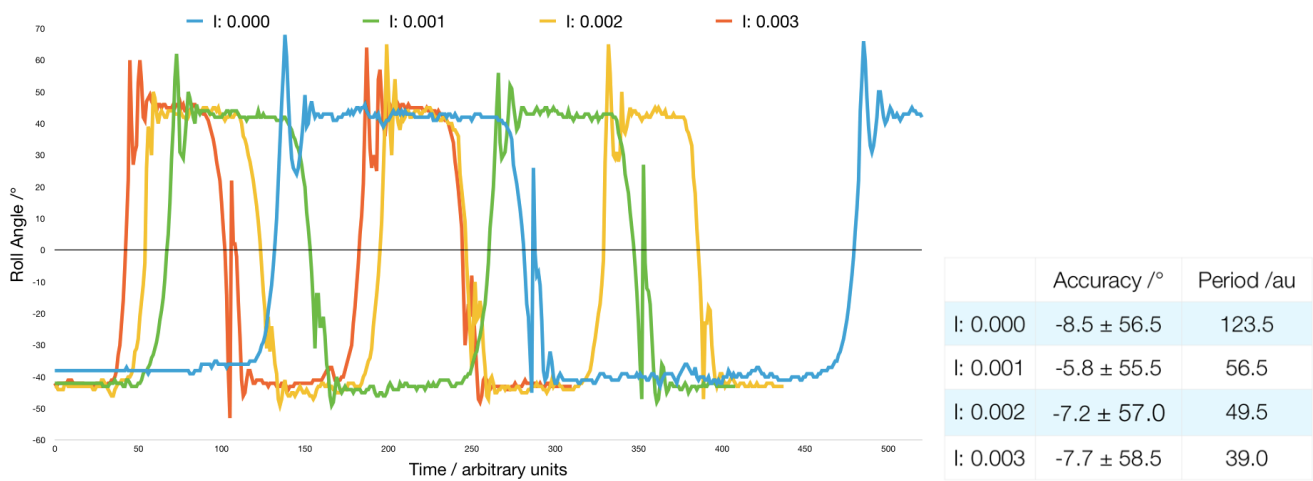


Figure 5: Comparison of different I-gain values on the stability of the quadcopter

As observed by the data recorded in the figure above, the period of oscillation required for the drone to correct its angle decreased when the I-gain increased from 0.000 to 0.004. Showing that the latency of the controller decreases as the I-gain increased. However, the accuracy did not increase as the quadcopter continued to overshoot.

This is expected from the integral term in the PID loop. The  $K_i \int_0^t e(t) dt$  term takes into account the previous errors and is a summation of the deviations of the quadcopter from its setpoint. Using this accumulation of error, it slowly produces a correction value to the actuators, which can be seen in the sharp corrections from  $-40^\circ$  to  $+40^\circ$ . Between the sharp corrections, the quadcopter slowly accumulates the error in the opposite direction again, and makes a correction. The term is especially good for correcting small systematic errors that the quadcopter may have, but is ineffective in accurately correcting the quadcopter by itself.

### 4.3 Comparison of D-gain on the stability of quadcopter

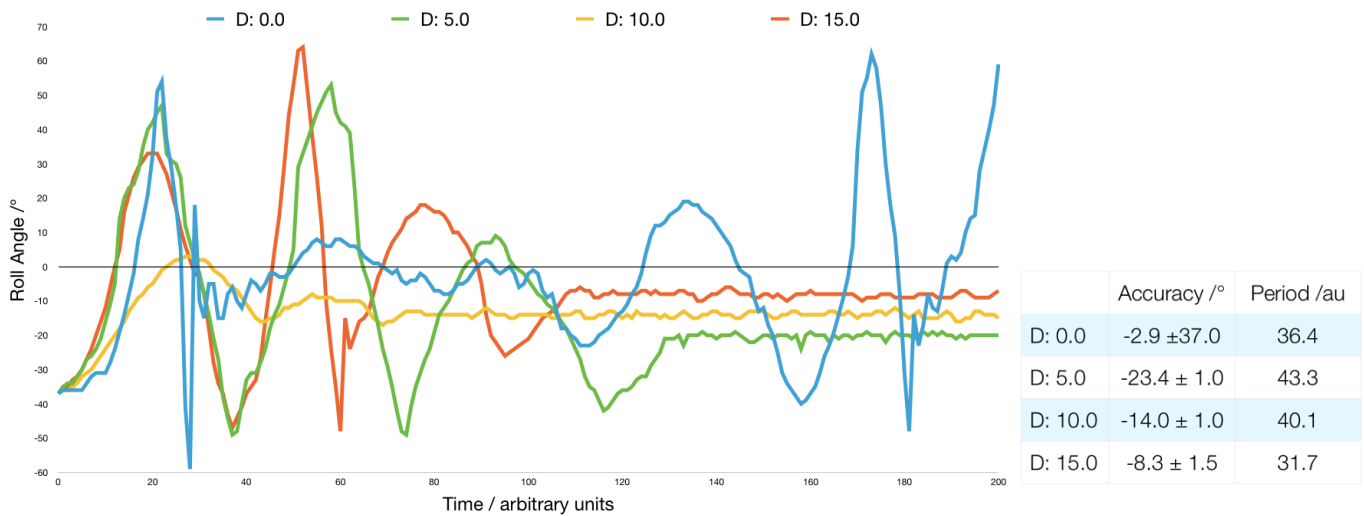


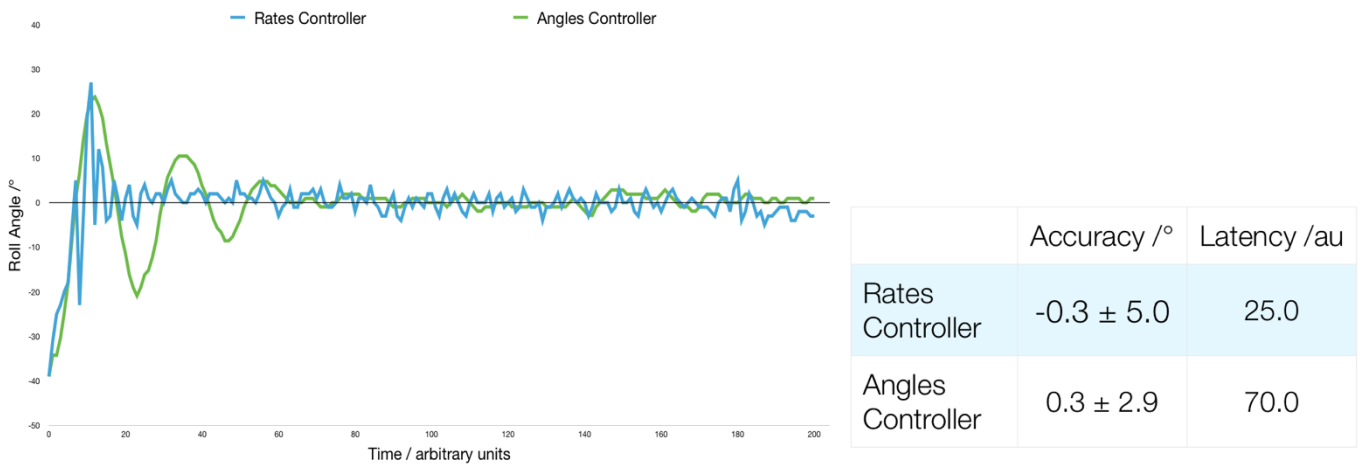
Figure 6: Comparison of different P-gain values on the stability of the quadcopter

From the data above, the quadcopter oscillated about the setpoint in an attempt to reach a roll angle of 0°. As the D-gain increased from 0.0 to 10.0, the amplitude of oscillation of the drone decreased. However, as the D-gain was increased from 10.0 to 15.0, the amplitude of oscillation increased. As the D-gain increased, the accuracy of the controller increased too. It was also observed that when the D-gain was 0, the quadcopter initially tried to correct its position but without the D-gain a small overcorrection causes the proportional controller to overcorrect the quadcopter and its amplitude of oscillation increased.

This can be explained by looking at the  $K_d \frac{de(t)}{dt}$  term in the PID loop. It acts as “resistance” towards change, and will attempt to reduce any change in orientation in the quadcopter. Thus, a higher D-gain slows down the correction in deviation of angle of the quadcopter by the P-gain, reaching the setpoint of 0° and reducing overshooting.

The period of the oscillations increased as the D-gain increased, implying the latency of the controller increased. As a higher D-gain would create higher “resistance” towards a correction in the deviation as explained earlier, it would increase the time required to reach the setpoint, hence the latency of the controller is increased. This is a trade-off that requires to be optimised to allow both accuracy in avoiding overcorrecting the quadcopter as well as not retaining the latency of the control system.

#### 4.4 Comparison of rates controller and angles controller



*Figure 7: Comparison between an angles controller and a rates controller on the stability of the quadcopter*

From the data above, the quadcopter with the angles controller was more accurate than the rates controller once it reached its setpoint. This is likely due to the fact that the angles data that is used in the calculations of the angles controller being filtered more times as it was calculated from the angular velocity data.

It was also observed that the rates controller had a lower latency than the angles controller. This could be because the angular velocity represents the rate of change of angle, and its value would mathematically change at a greater rate than the value of angles inputted into the controller.

#### 4. Conclusion

The Proportional-Integral-Derivative is a rudimentary yet complicated control theory that can be utilized to accurately control quadcopters with sufficient visualized knowledge on how to tune the respective gains of the controller. The angles controller was found to be relatively more accurate than the rates controller, while the rates controller had a lower latency in correction than the angles controller. Conventionally these controllers are used separately, with angles controller utilized when the objective of the quadcopter is to maintain stable flight, while the angles controller was utilized when the acrobatics of the quadcopter was demanded. It is also possible to develop a combined rates and angles controller that has both increased accuracy and reduced latency, allowing for a versatile quadcopter that is both stable and can perform acrobatics to be produced.



## 5. Bibliography

Atheer, L., Mahmoud, M., Mohamed, H., & Khalaf, G. (2010). *Flight PID controller design for a UAV quadrotor*. *Scientific Research and Essays*, 5(23), 3660–3667. doi: 1992-2248

Doitsidis, L., Valavanis, K., Tsourveloudis, N., & Kontitsis, M. (2004). *A framework for fuzzy logic based UAV navigation and control*. *IEEE International Conference on Robotics and Automation, 2004. Proceedings. ICRA 04. 2004*. doi: 10.1109/robot.2004.1308903

Fessi, R., & Bouallegue, S. (2016). *Modeling and Optimal LQG Controller Design for a Quadrotor UAV*. *Proceedings of Engineering and Technology*, 264–270. Retrieved from [http://ipco-co.com/PET\\_Journal/Acecs-2016/44.pdf](http://ipco-co.com/PET_Journal/Acecs-2016/44.pdf)

Gene H, McCall, John A, Corder (1997). *UAVs. New world vistas: Air and space for the 21st century*. *Human Syst. Biotechnol. Syst.*, (7):17–18.

Kada, B., & Ghazzawi, Y. (2011). *Robust PID Controller Design for an UAV Flight Control System*. *Proceedings of the World Congress on Engineering and Computer Science*, 2. doi: 978-988-19251-7-6

Pedley, M. (2013). *Tilt Sensing Using a Three-Axis Accelerometer*. *Freescale Semiconductors*, AN3461, 5–10. Retrieved from [https://cache.freescale.com/files/sensors/doc/app\\_note/AN3461.pdf](https://cache.freescale.com/files/sensors/doc/app_note/AN3461.pdf)

Stafford, J. (2014). *How a Quadcopter works* Clay Allen. *University of Alaska, Fairbanks*. Retrieved 2015-01-20.

Thamm, F.-P., Brieger, N., Neitzke, K.-P., Meyer, M., Jansen, R., & Mönninghof, M. (2015). *SONGBIRD – AN INNOVATIVE UAS COMBINING THE ADVANTAGES OF FIXED WING AND MULTI ROTOR UAS*. *ISPRS - International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, XL-1/W4, 345–349. doi: 10.5194/isprsarchives-xl-1-w4-345-2015